# Nonlinear filters for mitigation of man-made noise

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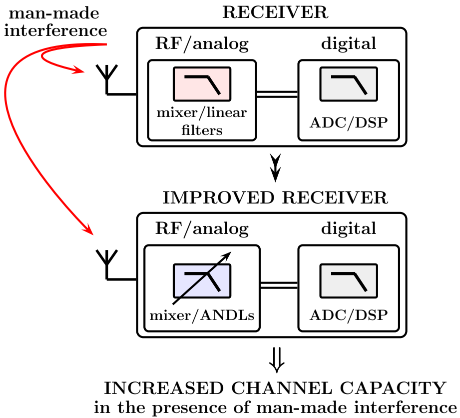
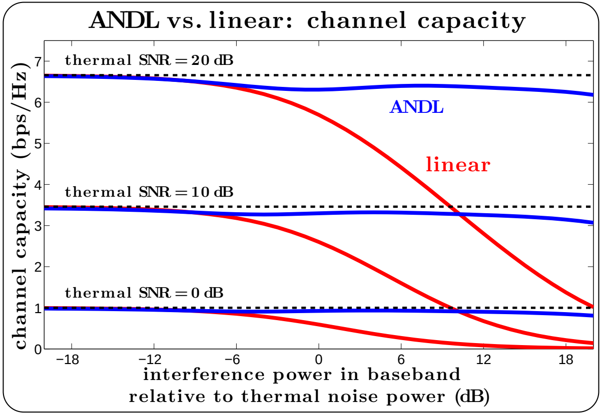
# Motivation

## Communications receivers resistant to technogenic interference




Replacing certain analog filters in communications receiver by ANDLs provides resistance to man-made interference

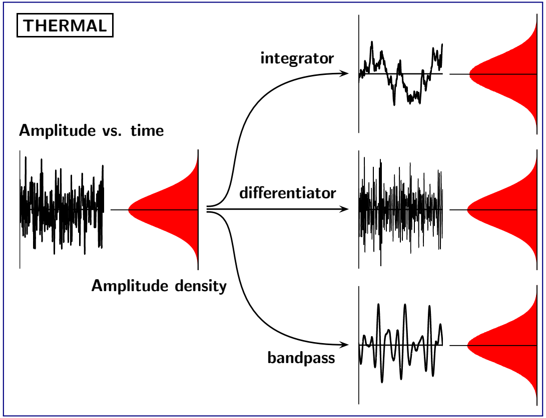
ANDLs vs. linear: baseband SNR (21/25)

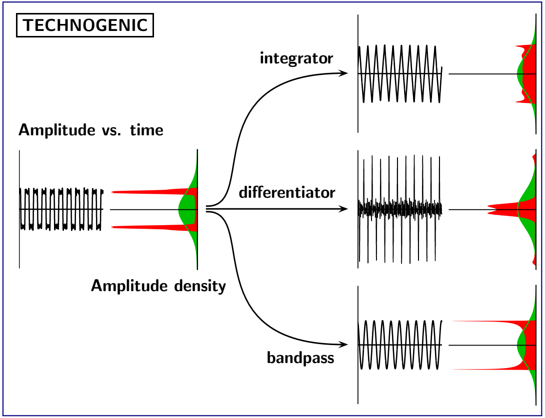
# Motivation

## Nonlinear vs. linear: The rationale

- Technogenic (man-made) signals are typically distinguishable from purely random (e.g. thermal)
  - specifically, in terms of amplitude distributions/densities (non-Gaussian)
- At any given frequency, linear filters affect power of both noise and signal of interest proportionally, and cannot improve SNR in passband
- Nonlinear filters can reduce PSD of non-Gaussian interference in passband without significantly affecting signals of interest
  - increasing passband SNR and channel capacity
- Linear filters are converted into Nonlinear Differential Limiters (NDLs) by introducing feedback-based nonlinearities into filter responses
  - NDLs/ANDLs are fully compatible with existing linear devices and systems
  - enhancements/low-cost alternatives to state-of-art interference mitigation methods

# Distributional differences between thermal noise and technogenic signals




For Gaussian (e.g. thermal) signals, amplitude distribution remains Gaussian regardless of linear filtering

Amplitude distributions of non-Gaussian signals are generally modifiable by linear filtering

# Distributional differences between thermal noise and technogenic signals

## Measures of peakedness

**Various measures/statistics can be used**

Measure of *peakedness* for $z(t) = I(t) + \mathrm{i}Q(t)$ used in this presentation:

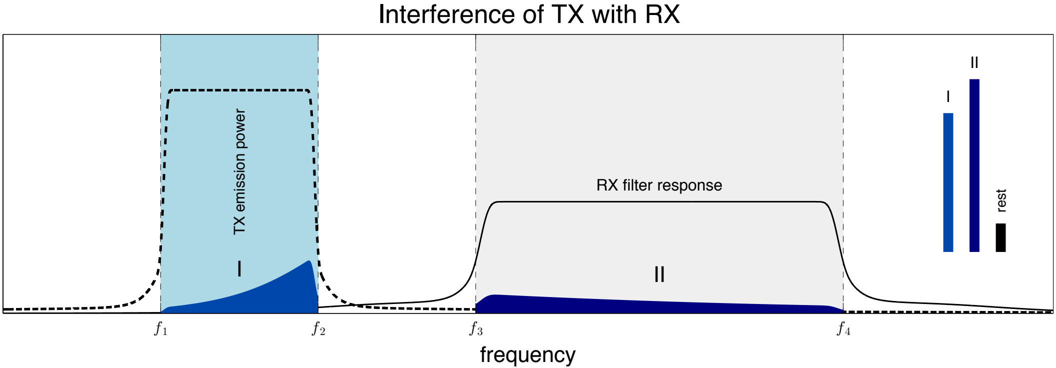
$$K_{\mathrm{dBG}}(z) = 10 \lg \left( \frac{\langle |z|^4 \rangle - |\langle zz \rangle|^2}{2\langle |z|^2 \rangle^2} \right)$$

- angular brackets denote time averaging
- based on definition of *kurtosis* for complex variables
- "decibels relative to Gaussian" (dBG) – in relation to Gaussian distribution
- $K_{\mathrm{dBG}}$ vanishes for Gaussian distribution
- $K_{\mathrm{dBG}} < 0$ for sub-Gaussian, $K_{\mathrm{dBG}} > 0$ for super-Gaussian
- high peakedness $\Rightarrow$ frequent occurrence of outliers (impulsive)

Average power and peakedness (10/25)

# Distributional differences between thermal noise and technogenic signals

## Impulsive nature of interchannel interference

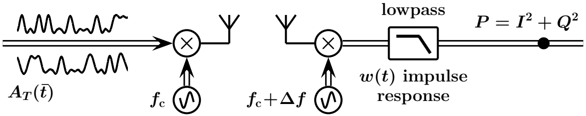



Qualitative illustration of different contributions into the interference which a receiver (RX) experiences from a transmitter (TX)

TX OOB interference in the RX channel (part II of the total interference) can appear impulsive under a wide range of conditions, and can degrade the RX communication signal as it raises the noise floor in the RX band

Impulsive interference (11/25)
Increasing peakedness (13/25)

# Impulsive nature of interchannel interference:
## TX OOB interference in the RX channel (part II of the total interference)




For example, it can appear as an impulsive pulse train

$$P(t, \Delta f) = \frac{1}{(T\,\Delta f)^{2n}} \sum_i |\alpha_i|^2\, h^2\left(\bar{t} - \bar{t}_i\right)$$

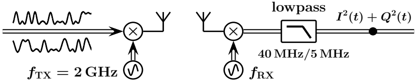
- for sufficiently large $T$ and/or $\Delta f$
- $T$ is symbol duration (unit interval)
- $\bar{t}$ is nondimensionalized time, $\bar{t} = \frac{2\pi}{T}t$
- $h(\bar{t}) = \frac{T}{2\pi} w(t)$, $w(t)$ is impulse response of lowpass filter
- $A_T(\bar{t})$ is modulating signal
- $|\alpha_i|$ is magnitude of discontinuity of $A_T^{(n-1)}(\bar{t})$ at $\bar{t}_i$

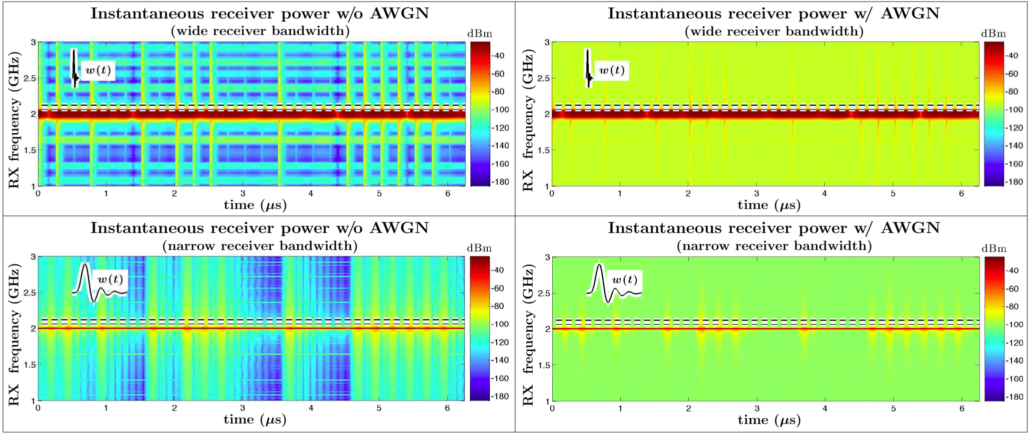
*EURASIP J Adv Signal Process* 2011, 2011:137
*Proc. IEEE Radio and Wireless Symposium*, Phoenix, AZ 2011:118-121
Experimental evidence: *EURASIP J Adv Signal Process* 2012, 2012:79

▸ Effects of symbol rates and pulse shaping (12/25)

# Impulsive nature of interchannel interference:

## Instantaneous power response of a quadrature receiver

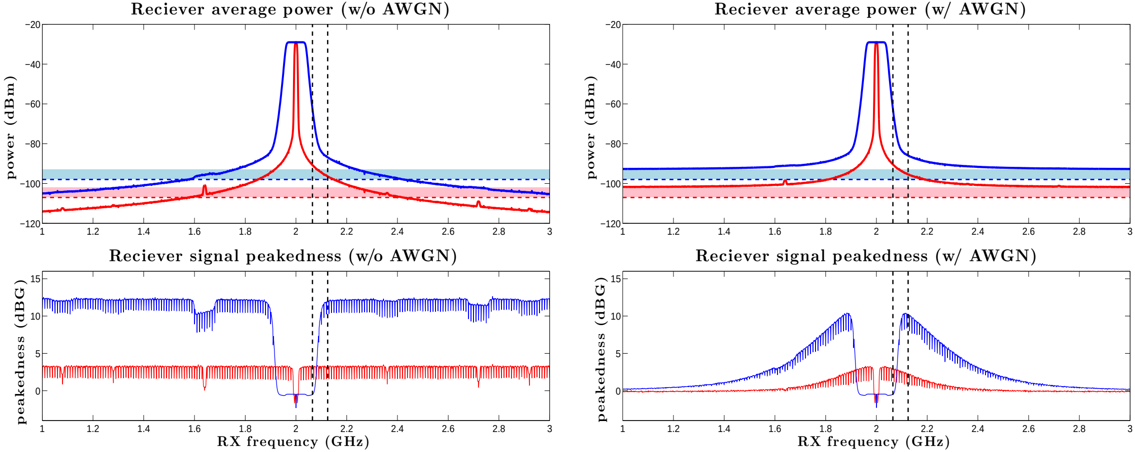



Simulation parameters (26/25)

# Impulsive nature of interchannel interference:

## Average power and peakedness



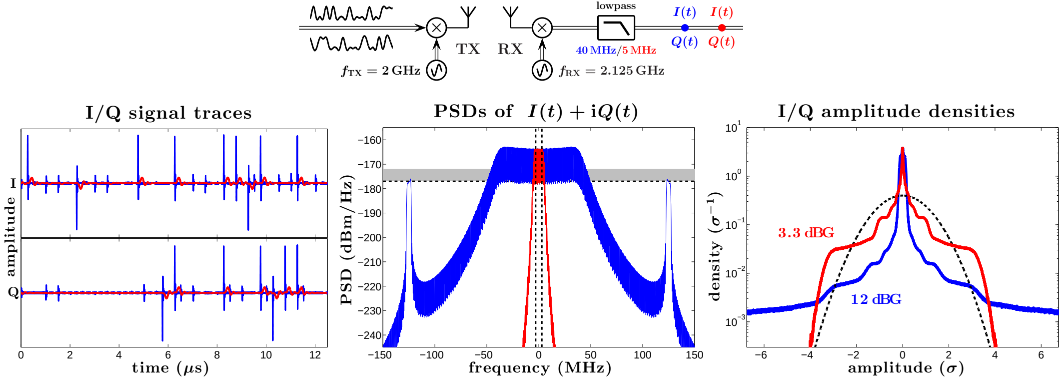

**Red lines:** 5 MHz lowpass | **Blue lines:** 40MHz lowpass

▸ Measure of peakedness (6/25)

▸ Simulation parameters (26/25)

# Distributional differences between thermal noise and technogenic signals

## Impulsive interference: Part II dominates



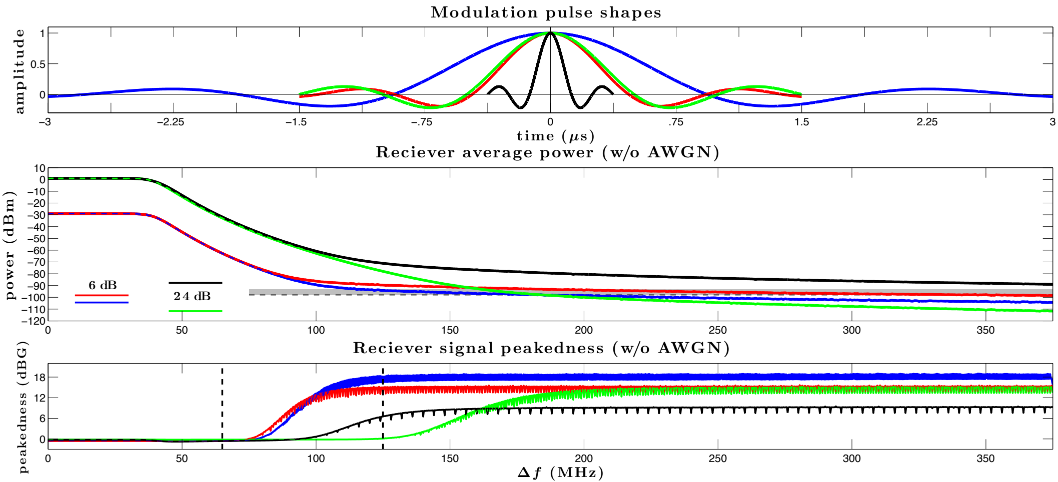

**Blue lines and text:** 40 MHz bandwidth **Red lines and text:** 5 MHz bandwidth

For a sufficiently large $|\Delta f|$ (e.g. 125 MHz), impulsive component (part II) dominates

TX RX interference (7/25) Simulation parameters (26/25)

# Impulsive nature of interchannel interference:

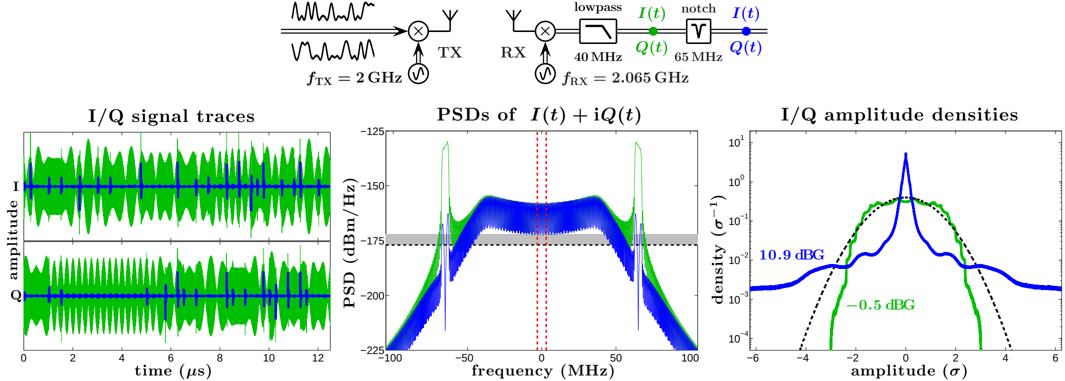
## Effects of symbol rates and pulse shaping on power and peakedness




OOB interference (8/25)
Simulation parameters (26/25)

# Distributional differences between thermal noise and technogenic signals

## Practical example of increasing peakedness



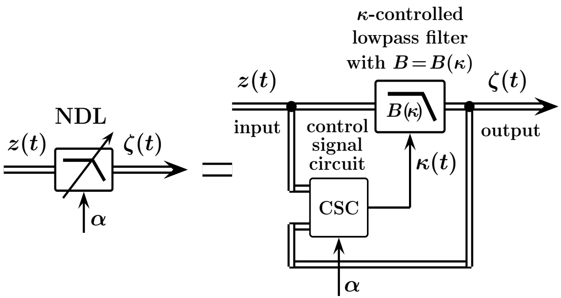

**Green lines and text:** before notch **Blue lines and text:** after notch

Notch filter reduces sub-Gaussian part of interference without affecting signal of interest and/or PSD of mpulsive interference around baseband, enabling its effective mitigation by NDLs

TX RX interference (7/25)
Peakedness and baseband SNRs (17/25)
Simulation parameters (26/25)

# Nonlinear Differential Limiters (NDLs)

NDLs are designed for mitigation of impulsive interference
(i.e. characterized by relatively high occurrence of outliers)




Block diagram of Nonlinear Differential Limiter

▸ Example of sub-circuit topologies (18/25)

- Dynamic modification of filter bandwidth based on magnitude of difference signal $z(t) - \zeta(t)$
- "Bandwidth" $B$ of NDL = bandwidth of linear filter with same coefficients
  - just convenient computational proxy
- $B$ is non-increasing function of $|z - \zeta|$
  - monotonically decreasing for $|z - \zeta| > \alpha$
  - $\alpha$ is resolution parameter
- Linear filter when $\alpha \to \infty$

# Nonlinear Differential Limiters (NDLs)

## 2nd order NDL example

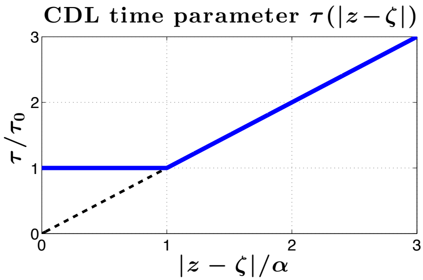
$\zeta(t) = z(t) - \tau\,\dot{\zeta}(t) - (\tau Q)^2\,\ddot{\zeta}(t)$ – second order linear lowpass filter

- $\tau$ is time parameter, $Q$ is quality factor
- "bandwidth" is *decreasing* function of $\tau$, *increasing* function of $Q$

**Constant-$Q$ NDL:**

$$\tau(|z-\zeta|) = \tau_0 \times \begin{cases} 1 & \text{for } |z-\zeta| \le \alpha \\ \left(\frac{|z-\zeta|}{\alpha}\right)^{\beta} & \text{otherwise} \end{cases}$$

- $\beta > 0$
- Canonical Differential Limiter (CDL) for $\beta = 1$
- Differential over-Limiter (DoL) for $\beta > 1$

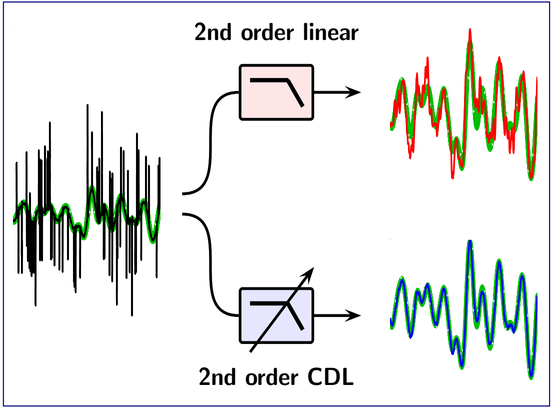


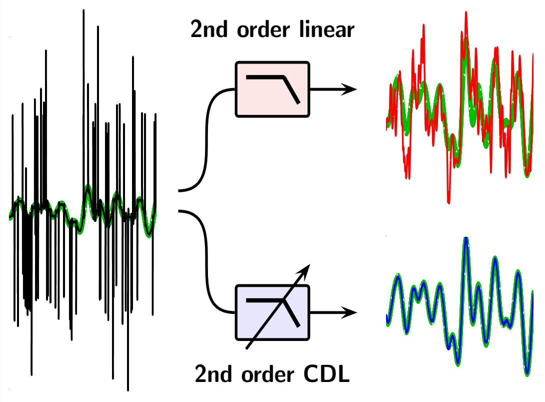

More on NDLs: US patent 8,489,666 (16 July 2013) / US patent application publication 2013/0339418 (Dec. 19, 2013)

# Nonlinear Differential Limiters (NDLs)

## 2nd order CDL: Nonlinear suppression of impulsive noise

"Disproportional" (nonlinear) suppression of impulsive noise by NDLs

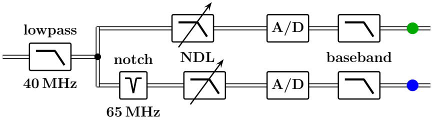


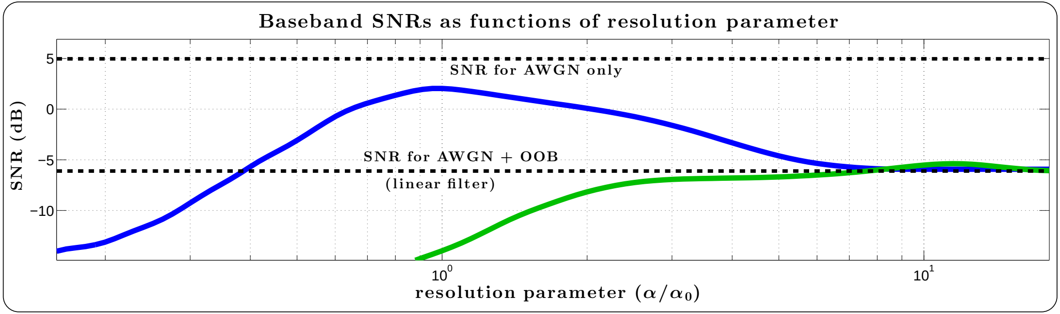




Linear filter: Output noise is proportional to input noise NDL: Output is insensitive to impulsive noise

# NDL-based mitigation of out-of-band interference:

## SNRs in the receiver as functions of the NDL resolution parameter




**Green:** w/o notch **Blue:** with notch

▸ Increasing peakedness (13/25) ▸ Simulation parameters (26/25)

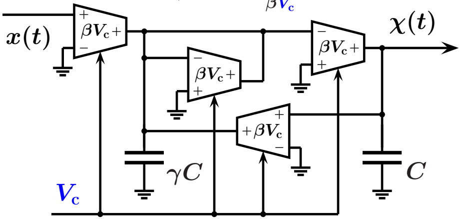
# Nonlinear Differential Limiters (NDLs)

## Example of sub-circuit topologies

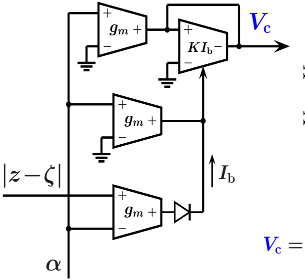
### OTA-based 2nd order CDL

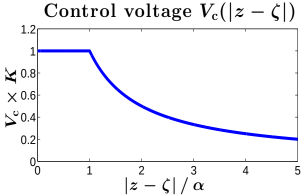
$$\chi(t) = x(t) - \tau\,\dot{\chi}(t) - (\tau Q)^2\,\ddot{\chi}(t)$$

$$Q = \sqrt{\gamma}\,, \quad \tau = \frac{C}{\beta V_c}$$




Voltage-controlled 2nd order lowpass filter with Tow-Thomas topology




$$V_c = \frac{1}{K} \times \begin{cases} 1 & \text{for } |z-\zeta| \le \alpha \\ \frac{\alpha}{|z-\zeta|} & \text{otherwise} \end{cases}$$

CDL control voltage circuit

▸ Nonlinear Differential Limiters (14/25)

# Adaptive NDLs (ANDLs)

An ANDL contains a sub-circuit (characterized by a *gain* parameter) that monitors a chosen measure of the signal+noise mixture and provides a time-dependent resolution parameter $\alpha = \alpha(t)$ to the main NDL circuit

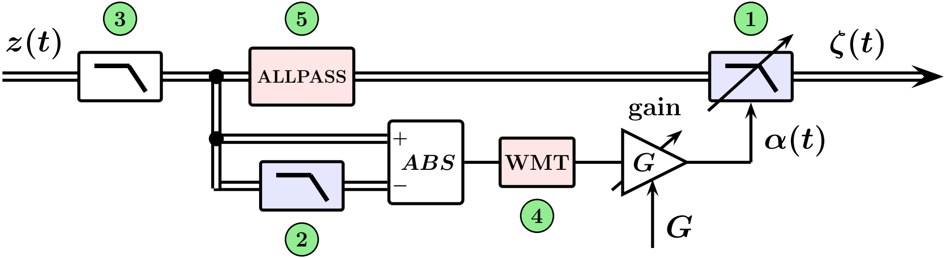
- suitable for improving quality of non-stationary signals under time-varying noise conditions

More on NDLs/ANDLs:

- **Method and apparatus for signal filtering and for improving properties of electronic devices.** US patent application publication 2013/0339418 (Dec. 19, 2013)
- **Adaptive analog nonlinear algorithms and circuits for improving signal quality in the presence of technogenic interference.** In Proc. IEEE Military Communications Conference (MILCOM 2013), San Diego, CA, 18-20 November 2013
- http://www.avatekh.com

# Adaptive NDLs (ANDLs)

## ANDL example

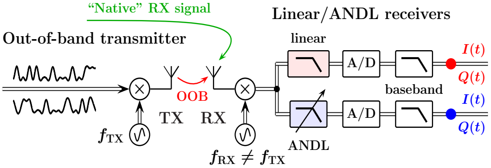


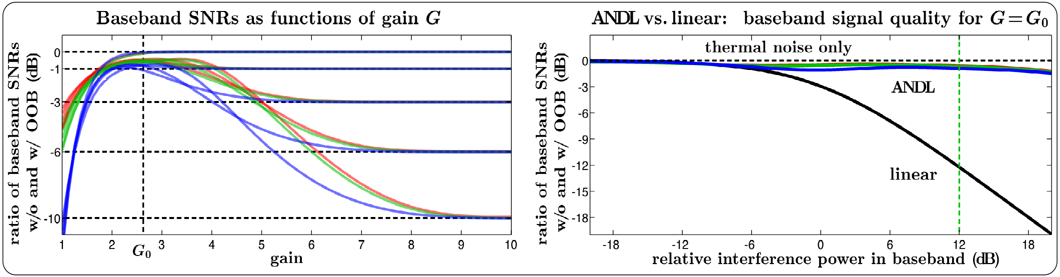

1. 2nd order constant-$Q$ DoL ($\beta = 2$) with $\tau_0 = \frac{1}{2\pi f_0}$
2. 2nd order lowpass with $\tau = \tau_0$ (same $Q$)
3. Higher-order lowpass with $\tau \ll \tau_0$
4. WSMR circuit w/ 2nd order Bessel window ($\tau_b = 2\tau_0/\sqrt{3}$, $Q = 1/\sqrt{3}$)
5. Allpass with delay $2\tau_0$

▸ Digital ANDL (24/25)

# Adaptive NDLs (ANDLs)

## ANDLs at work



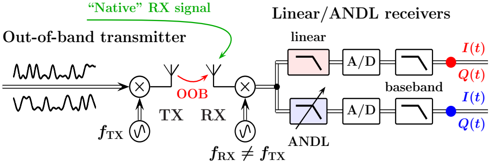



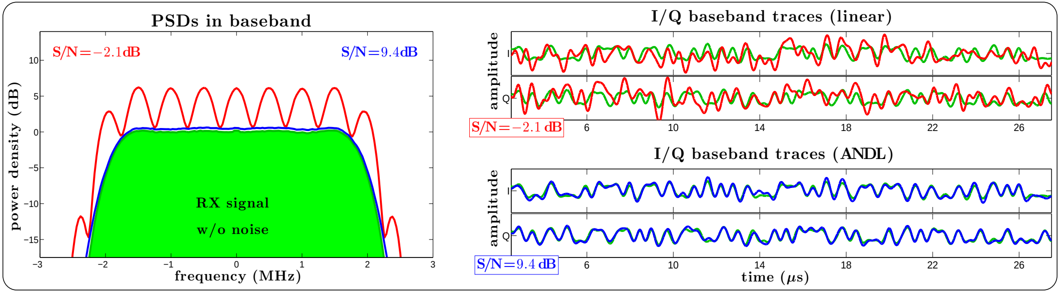

RED: 0 dB thermal SNR GREEN: 10 dB thermal SNR BLUE: 20 dB thermal SNR

▸ Communications receivers resistant to man-made interference (3/25)

# Adaptive NDLs (ANDLs)

## ANDLs at work



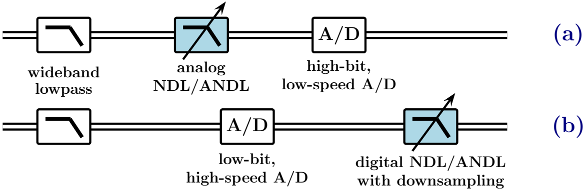




**LEFT:** Interference PSD reduction in passband **RIGHT:** Baseband time domain I/Q traces

# Digital NDLs/ANDLs

## Digital NDLs/ANDLs

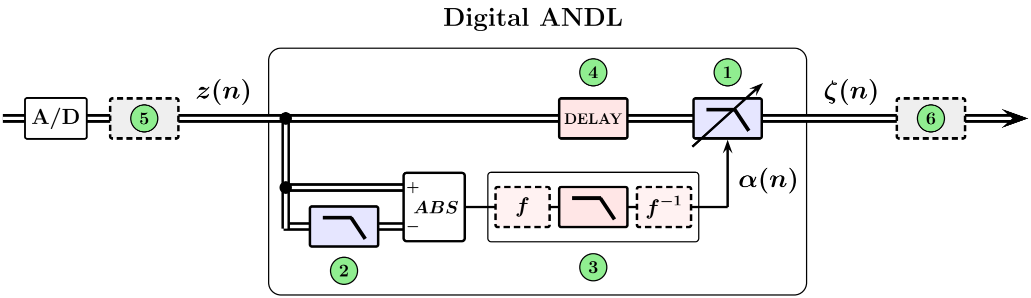
- NDLs/ANDLs are *analog* filters
  - combine bandwidth reduction with mitigation of interference
- Also allow for near-real-time finite-difference (digital) implementations
  - relatively simple
  - computationally inexpensive
  - low memory requirements
- Digital NDLs/ANDLs require high sampling rates
  - should use multi-rate processing




Analog (a) and digital (b) NDL/ANDL deployments

# Digital NDLs/ANDLs

## Digital ANDL example

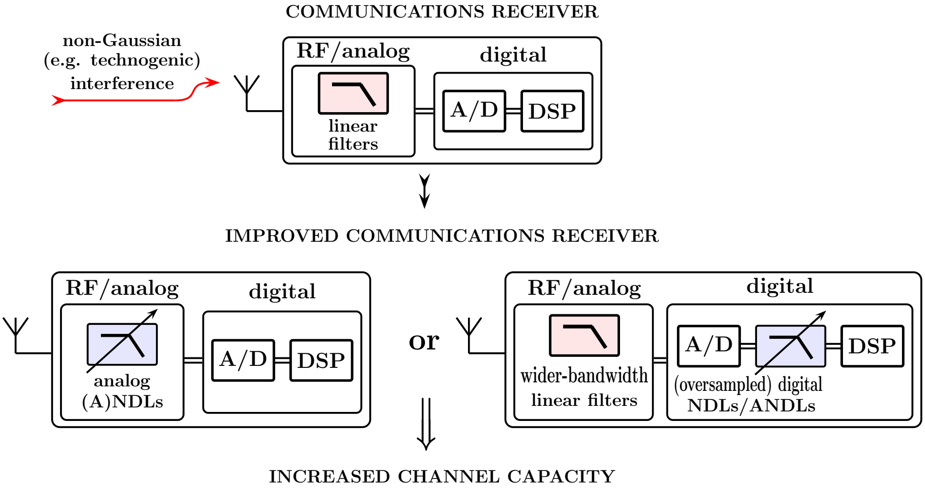



1. Digital NDL
2. Linear lowpass filter equivalent to NDL with $\alpha \to \infty$
3. WMT module
4. Digital delay line (to compensate for WMT delay)
5. Optional linear filter to increase peakedness (e.g. notch)
6. Equalization / decimation

Analog ANDL (20/25)

# Digital NDLs/ANDLs

## Communications receivers resistant to technogenic interference



# Appendix I: Simulation parameters

The TX signal used in the simulations on pages 9–13 and 17 was a random QPSK signal. In all simulations except those shown on page 12 the symbol rate was 4 Mbit/s (unit interval $T = 250$ ns), and an FIR RRC filter with the roll-off factor 1/4 and the group delay $3T$ was used for pulse shaping. The average TX signal power was 125 mW (21 dBm), and the path/coupling loss at any RX frequency was 50 dB, except for the TX signals shaped with the filters shown by the black and green lines on page 12, where it was 20 dB

The RX lowpass filters were 8th order Butterworth filters. A 5 dB noise figure of the receiver was assumed at all receiver frequencies $f_{\mathrm{RX}}$ ($\Rightarrow -172$ dBm/Hz for the total AWGN level at room temperature). The incoming RX signal used on page 17 was a random QPSK signal with the rate 4.8 Mbit/s. An FIR RRC filter with the roll-off factor 1/4 and the group delay $3T$ was used for the RX incoming signal pulse shaping, and the same FRI filter was used for the matched filtering in the baseband. The PSD of the RX signal without noise was approximately $-167$ dBm/Hz in the baseband, leading to the S/N ratio without interference of approximately 5 dB

▸ Instantaneous power response of a quadrature receiver (9/25)
▸ Average power and peakedness (10/25)
▸ Impulsive interference: Part II dominates (11/25)
▸ Effects of symbol rates and pulse shaping (12/25)
▸ Practical example of increasing peakedness (13/25)
▸ Peakedness and baseband SNRs (17/25)

# Appendix II: References to relevant work


- Nikitin AV
  *Method and apparatus for signal filtering and for improving properties of electronic devices*
  US Patent Application Publication 2013/0339418 (9 December 2013)
- Nikitin AV, Davidchack RL, Sobering TJ
  Adaptive analog nonlinear algorithms and circuits for improving signal quality in the presence of technogenic interference
  In *Proc. IEEE Military Communications Conference (MILCOM 2013)*, San Diego, CA, 18-20 November 2013
- Nikitin AV, Davidchack RL, Smith JE
  Out-of-band and adjacent-channel interference reduction by analog nonlinear filters
  In *Proc. of the 3rd IMA Conference on Mathematics in Defence*, Malvern, UK, 24 October 2013
- Nikitin AV
  *Method and apparatus for signal filtering and for improving properties of electronic devices*
  US Patent 8,489,666 (July 16, 2013)
- Nikitin AV, Epard M, Lancaster JB, Lutes RL, Shumaker EA
  Impulsive interference in communication channels and its mitigation by SPART and other nonlinear filters
  *EURASIP Journal on Advances in Signal Processing*, 2012, 2012:79
- Nikitin AV
  On the interchannel interference in digital communication systems, its impulsive nature, and its mitigation
  *EURASIP Journal on Advances in Signal Processing*, 2011, 2011:137
- Nikitin AV
  On the impulsive nature of interchannel interference in digital communication systems
  In *Proc. IEEE Radio and Wireless Symposium*, Phoenix, AZ 2011:118-121

# Appendix III: Disclaimer

Part of the material contained in this presentation is based upon work supported by the National Science Foundation under Grant Number 1314790. However, any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation